Question 1: **[2 points]**

a) Let L be a language over the alphabet Σ, and define L^+ to be the language $\bigcup_{k\geq 1} L^k$. Clearly, from the definition, we have that $L^+ \subseteq L^*$. Under which circumstances are they equal? Prove you claim.

b) Five two languages L_1 and L_2 over the alphabet {a,b} such that $L_1L_2 = L_2L_1$, both are different from $\{\Lambda\}$, and neither language contains the other one.

Question 2: **[4 points]**

Let $M = (Q,\Sigma, q_0,A,\delta)$ be a FA with a state $q \in Q$ such that that $\delta(q,\sigma) = q$ for all $\sigma \in \Sigma$. This means that whatever the input symbol, if M is in state q it will remain there. Use induction to prove that once in q, M will never leave that state anymore whatever the input string it is being fed, i.e. prove that $\delta^*(q, x) = q$ for all $x \in \Sigma^*$

Question 3: **[4 points]**

a) Draw a finite automaton M accepting the language $L = \{ x \in \{a,b\}^* \mid x$ ends with aa or with b$\}$.

b) Find three distinct strings x and y and z in {a, b}* that are pairwise L distinguishable.

c) Find two distinct strings x and y in {a, b}* that are not L distinguishable.

d) Give an automaton recognizing the language $S = \{ x \in \{a,b\}^* \mid x$ ends with a$\}$, and using your automaton M, construct an automaton recognizing the language $L \cap S$.